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Integer programming for optimized nurse scheduling: A model for efficient workforce allocation in healthcare systems

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Abstract

Effective nurse scheduling is essential for optimizing healthcare workforce management. This study presents an integer programming model that ensures optimal shift allocation while balancing operational efficiency, legal constraints, and staff preferences. By minimizing scheduling inefficiencies and improving workload distribution, the model enhances both cost-effectiveness and nurse satisfaction. The result of implementation at mile four hospital Abakaliki, Nigeria produces optimal allocation of nurses for days off and also optimal number of nurse requirement for each ward or unit. Results demonstrate superior performance over traditional scheduling methods by minimizing 0.09% of the hospital's total nursing staff cost. This framework offers a scalable solution for improving workforce allocation in healthcare systems.

Keywords: Nurse Scheduling; Integer Programming; Workforce Optimization; Healthcare Operations

1. Introduction

A mathematical programming approach has been shown to effectively minimize nursing shortages and satisfy staffing constraints (Warner and Prawda, 1972).

Abdalkareem et al. (2021) provided a comprehensive survey of healthcare scheduling problems, highlighting key areas such as nurse scheduling, patient admissions, and operating room planning, and analyzing 190 articles across various optimization approaches.

Efficient nurse scheduling is fundamental to the smooth operation of healthcare facilities, directly impacting patient care, hospital efficiency, and staff well-being. Given the critical role that nurses play in patient management, ensuring adequate staffing levels while balancing nurse workload, regulatory requirements, and institutional constraints remains a complex challenge. The Nurse Scheduling Problem (NSP) is a well-documented issue in healthcare operations, requiring hospitals to develop shift allocations that minimize inefficiencies, prevent staff burnout, and optimize resource utilization. An effective scheduling system ensures uninterrupted patient care, adherence to labor laws, and fair workload distribution among nurses.

Traditionally, hospitals have relied on manual scheduling methods, which are often time-consuming, error-prone, and inflexible. These traditional approaches struggle to accommodate fluctuating patient demands, staff preferences, and legal constraints, leading to understaffing or overstaffing, increased overtime costs, and nurse dissatisfaction. Studies have shown that poor scheduling contributes to low job satisfaction and high turnover rates, which in turn affect the quality of healthcare delivery (Cheang et al., 2003; Burke et al., 2004). Additionally, ineffective scheduling can result in higher operational costs, as hospitals often rely on expensive temporary staffing solutions to fill gaps in coverage (Smith

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and Wiggins, 1997). These challenges necessitate the adoption of more systematic and data-driven approaches to workforce scheduling.

In response to these inefficiencies, Integer Programming (IP) has emerged as a powerful optimization tool for solving complex scheduling problems, including NSP. Unlike heuristic or manual methods, IP ensures mathematically optimal solutions by systematically assigning nurses to shifts while satisfying multiple constraints. This approach enhances fairness, efficiency, and cost-effectiveness, ensuring compliance with institutional policies and labor laws. Prior research has demonstrated that IP-based scheduling models improve nurse allocation, enhance workforce efficiency, and reduce scheduling conflicts in hospital settings (Alfares, 2002; El-Quliti and Al-Darrab, 2009). By leveraging mathematical optimization, hospitals can achieve a balance between workforce demands and staff well-being, leading to improved patient care outcomes.

This study applies an Integer Programming model to optimize nurse scheduling at Mile Four Hospital, Abakaliki, which operates 21 wards with a total of 64 full-time nurses. Given that each nurse is required to work five days per week with two consecutive days off, scheduling becomes a highly constrained problem that requires careful optimization. The study aims to develop a structured, data-driven model that efficiently assigns nurses to shifts while ensuring compliance with hospital policies and staffing requirements. The key objectives of this research are to: Develop an optimized nurse scheduling model that ensures proper shift coverage while adhering to staffing requirements. Minimize inefficiencies such as understaffing, excessive overtime, and workload imbalances. Enhance nurse satisfaction by promoting fair workload distribution. Ensure regulatory compliance, particularly in terms of maximum working hours and mandated rest periods.

By applying Integer Programming, this research contributes to the field of healthcare operations management, offering a scalable and decision-support framework for hospital administrators seeking to improve workforce planning. The findings will not only benefit Mile Four Hospital but can also serve as a benchmark for optimizing nurse scheduling in other healthcare institutions. Through this study, hospitals can achieve a balance between cost-effectiveness, staff well-being, and patient care quality, reinforcing the significance of mathematical optimization in modern healthcare management.

2. Methodology

2.1. Problem Formulation

The Nurse Scheduling Problem (NSP) is a highly constrained combinatorial optimization problem, where the goal is to create an efficient roster that assigns nurses to shifts while satisfying multiple constraints. The primary challenge in nurse scheduling is balancing hospital staffing requirements, employee preferences, legal regulations, and operational efficiency.

To address this, the problem is formulated as an Integer Programming (IP) model, which ensures mathematically optimal solutions while adhering to defined constraints. The scheduling problem involves allocating shifts to nurses while ensuring adequate shift coverage (meeting daily nurse demand per ward), fair workload distribution (avoiding nurse burnout and excessive overtime), regulatory compliance (adhering to labor laws and hospital policies). days-off scheduling constraints (ensuring every nurse gets two consecutive days off per week).

This study focuses on days-off scheduling, where the decision variables represent the assignment of nurses to specific off-day patterns. The IP model minimizes scheduling inefficiencies while satisfying all hospital constraints.

2.2. Mathematical Model

The NSP is formulated as an Integer Linear Programming (ILP) model, which ensures that the number of nurses assigned to each shift is an integer value. The key components of the model are:

2.3. Decision Variables

Let X_i=Number of nurses assigned to a specific days-off pattern (i=1,2, 7)

- X_1: Saturday-Sunday off
- X_2: Sunday-Monday off
- X_3: Monday-Tuesday off

- X_4: Tuesday-Wednesday off
- X_5: Wednesday-Thursday off
- X_6: Thursday-Friday off
- X_7: Friday-Saturday off

2.4. Objective Function

The goal is to minimize the total number of days-off allocations while ensuring that each ward meets its daily staffing requirements.

Min $Z=X_1+X_2+X_3+X_4+X_5+X_6+X_7$

2.5. Constraints

Each day must meet a minimum staffing requirement b_j (nurses needed per shift per ward). The model ensures that enough nurses are scheduled on duty each day:

- $X_1+X_2+X_3+X_4+X_5 \geq b_1$ (Saturday)
- $X_2+X_3+X_4+X_5+X_6 \geq b_2$ (Sunday)
- $X_3+X_4+X_5+X_6+X_7 \geq b_3$ (Monday)
- $X_1+X_4+X_5+X_6+X_7 \geq b_4$ (Tuesday)
- $X_1+X_2+X_5+X_6+X_7 \geq b_5$ (Wednesday)
- $X_1+X_2+X_3+X_6+X_7 \geq b_6$ (Thursday)
- $X_1+X_2+X_3+X_4+X_7 \geq b_7$ (Friday)

Additionally, the number of assigned nurses must be non-negative integers

$X_i \geq 0$, X_S is an integer for all i.

2.6. Solution Approach (Branch and Bound Algorithm)

2.6.1. Step 1: Solve the Linear Relaxation (LP Relaxation)

- Ignore integer constraints and solve the linear programming (LP) version of the problem using the Simplex Method.
- The solution obtained will have fractional values for some variables, which are not feasible since nurses cannot be assigned in fractional numbers.
- If the LP solution is already an integer, it is the optimal solution. Otherwise, proceed to branching

2.6.2. Step 2: Branching (Divide the Problem into Sub problems)

- Identify the first non-integer variable in the LP solution.
- Create two new sub problems (branches) by forcing the non- integer variable to take integer values.
- These two constraints create two new LP problems, each with a smaller feasible region.

2.6.3. Step 3: Bounding (Eliminating Non-Optimal Solutions)

- Solve both sub problems using the LP relaxation.
- If a branch leads to an infeasible solution (i.e., violates nurse staffing constraints), it is pruned (discarded).
- If a branch gives an integer solution, record it as a candidate optimal solution.
- If the objective function value of a new integer solution is worse than an already known feasible solution, discard it

2.6.4. Step 4: Node exploration

- Select the next non-integer variable and repeat the branching and bounding process.
- If a branch leads to a better feasible integer solution, update the best-known solution.
- Stop when all branches have either been explored or pruned.
- The best feasible integer solution found is the optimal nurse schedule.

2.7. Data Collection and Representation

This study is based on real hospital data collected from Mile Four Hospital, Abakaliki, Ebonyi State, Nigeria. The hospital has 21 yards and 64 full-time nurses, each required to work five days per week with two consecutive days off.

2.7.1. Ward-wise Staffing Data

The table below presents the total nurse availability and the daily staffing requirement per ward

Table 1 Nurses distribution per ward

Ward/Unit	Total Nurses Available	Daily Requirement
Postnatal (1) and (2)	9	4
Postnatal (3)	6	2
Nursery	6	4
Labour Ward	12	6
Antenatal Clinic (ANC)	6	6
Anesthesia Unit (ART)	2	2
Multiple Drug Resistant (MDR)	2	2
Outpatient Dept. (OPD)	3	2
Children's Ward	8	4
Operating Theatre	2	1
Admin/Counseling Unit	2	2
Tuberculosis Ward	1	1

2.8. Constraint Representation for Each Ward

For each ward/unit, we formulate separate constraints using their daily staffing requirements.

2.8.1. Formulation for Postnatal (1) and (2) Ward

This ward requires 4 nurses per day thus, the constraints specific to this ward will be:

- $X_1+X_2+X_3+X_4+X_5\geq4$ (Saturday)
- $X_2+X_3+X_4+X_5+X_6\geq4$ (Sunday)
- $X_3+X_4+X_5+X_6+X_7\geq4$ (Monday)
- $X_1+X_4+X_5+X_6+X_7\geq4$ (Tuesday)
- $X_1+X_2+X_5+X_6+X_7\geq4$ (Wednesday)
- $X_1+X_2+X_3+X_6+X_7\geq4$ (Thursday)
- $X_1+X_2+X_3+X_4+X_7\geq4$ (Friday)

Similar constraints apply for all other wards, adjusting b_j for each day's specific staffing needs.

The above is represented in the following matrix form

$$\text{Minimize } z = [1\ 1\ 1\ 1\ 1\ 1\ 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix}$$

$$\text{Subject to: } \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \geq \begin{bmatrix} 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \end{bmatrix}$$

Figure 1 Integer linear programming model for nurses distribution per ward

Similarly, integer programming formulation for F2 representing Anesthesia unit, Counseling unit and Multiple drug-resistant units with a daily requirement of 2 nurses, F3 representing Labor ward and Antenatal clinic with a daily requirement of 6 nurses, and F4 representing Operating theatre and Tuberculosis ward with daily requirement of 1 nurse were formulated with values of b_j's on the right-hand side of the constraints as 2, 6 and 1 respectively.

3. Result analysis

The excel solver was used to execute the branch and bound algorithm and the results are shown in the tables and figures below.

Table 2 LP Solution for F1

	X1	X2	X3	X4	X5	X6	X7		
Objective	1	1	1	1	1	1	1	6	
Constraint 1	1	1	1	1	1	0	0	4	4
Constraint 2	0	1	1	1	1	1	0	6	4
Constraint 3	0	0	1	1	1	1	1	4	4
Constraint 4	1	0	0	1	1	1	1	4	4
Constraint 5	1	1	0	0	1	1	1	4	4
Constraint 6	1	1	1	0	0	1	1	4	4
Constraint 7	1	1	1	1	0	0	1	4	4
Bound	0	0	0	0	0	0	0		
Decisions	0	2	0	2	0	2	0		

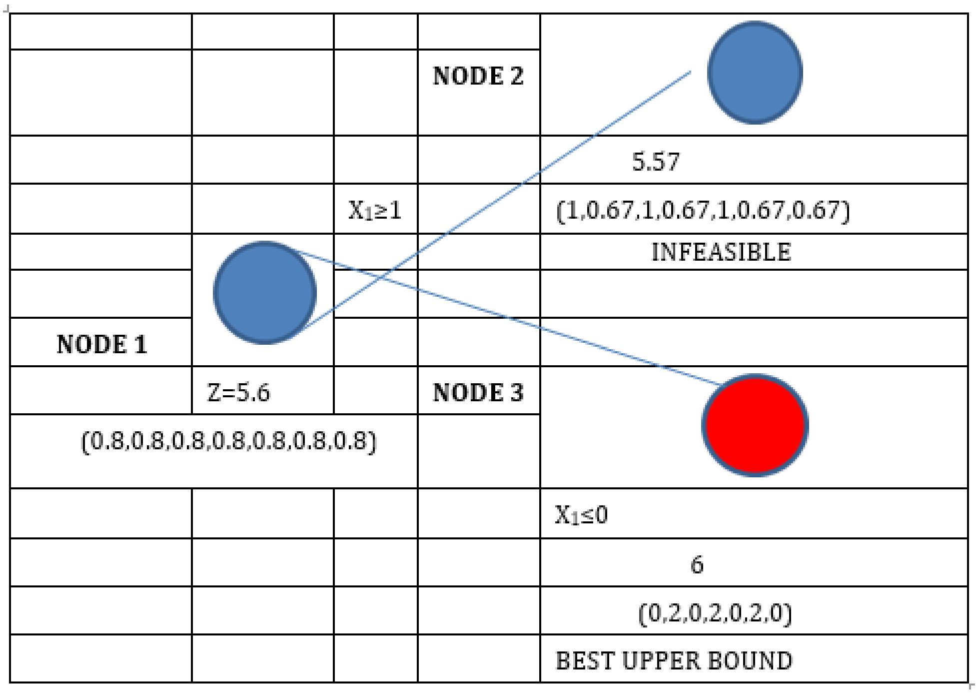
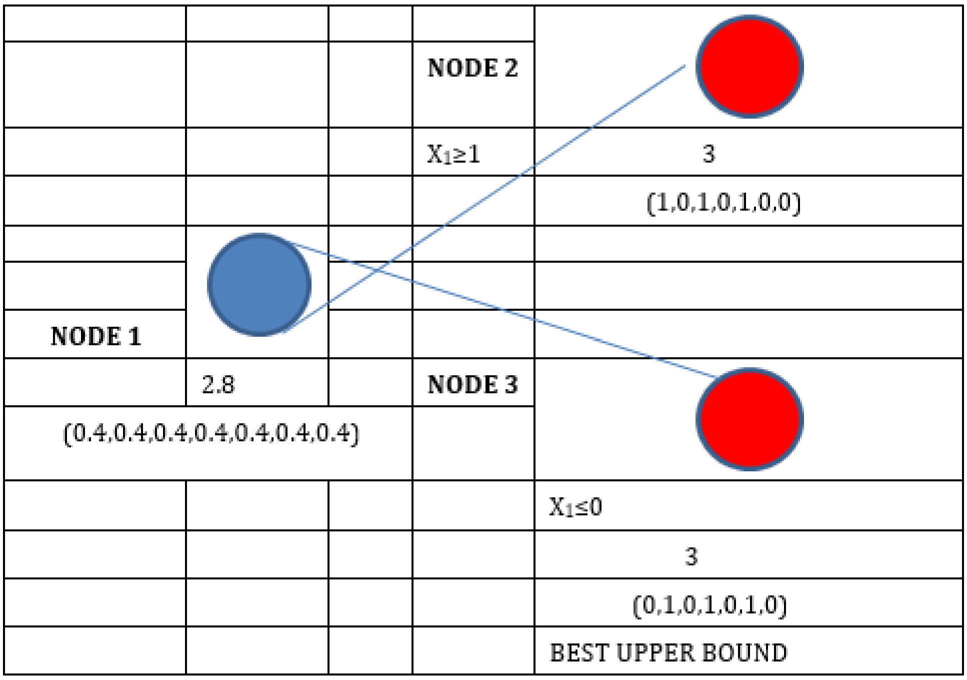


Figure 2 Branch and Bound Enumeration Tree for solution of F1

Table 3 LP solution for F2

	X_1	X_2	X_3	X_4	X_5	X_6	X_7		
Objective	1	1	1	1	1	1	1	3	
constraint 1	1	1	1	1	1	0	0	2	2
constraint 2	0	1	1	1	1	1	0	3	2
constraint 3	0	0	1	1	1	1	1	2	2
constraint 4	1	0	0	1	1	1	1	2	2
constraint 5	1	1	0	0	1	1	1	2	2
constraint 6	1	1	1	0	0	1	1	2	2
constraint 7	1	1	1	1	0	0	1	2	2
Bound	0	0	0	0	0	0	0		
Decisions	0	1	0	1	0	1	0		



Similarly, the objective value(minimum) for F3 consists of 7 nodes with Z =9, while the objective value for F4 consists Best of 3 nodes with Z= 1.5. The table below summarizes the optimal solution of the nurse scheduling problem for each grouping.

Figure 3 Branch and Bound Tree for Enumeration solution of F2

Table 4 Summary of the optimal solution of the Nurse Scheduling problem

Group	X_1	X_2	X_3	X_4	X_5	X_6	X_7	Objective value (z)
F1	0	2	0	2	0	2	0	6
F2	0	1	0	1	0	1	0	3
F3	1	1	1	1	2	1	2	9
F4	1	0	0	0	1	0	0	2

4. Discussion of Results

Based on the analysis we obtained the following key findings

In the post-natal (1) and (2) ward, Nursery ward and Children ward (F1) which has total availability of 9,6 and 8 employed nurses respectively; (see Table 1), an optimal total of 6 nurses for each of the wards in this group (F1) is needed in order to satisfy their daily requirement of 4 nurses. Also, x2 =2, x4 =2, x6 =2 indicates that two nurses should be assigned to Sunday – Monday off, Tuesday – Wednesday off, Thursday – Friday off respectively; (see table 7), x1=0, x3=0, x5=0, x7=0 implies that no nurse should be assigned to Saturday –Sunday off, Monday –Tuesday off, Wednesday –Thursday off and Friday –Saturday off.

In post-natal (3) ward, Anesthesia unit (A R T), Multiple drug-resistant units and out–patients department (F2) which has total of availability of 6,6,2 and 3 employed nurses respectively; (see table 1), an optimal total of 3 nurses for each of the wards/unit in this group (F2) is needed in order to satisfy their daily requirement of 2 nurses. Also, x2=1, x4=1, x6=1 indicates that one nurse should be assigned to Sunday –Monday off

Tuesday –Wednesday off, and Thursday –Friday off respectively. (see table 8) Then, x1=0 x3=0 x5=0 and x7=0 implies that no nurse should be assigned to Saturday – Sunday off, Monday –Tuesday off, Wednesday-Thursday off and Friday-Saturday off.

In labour ward and Antenatal clinic (F3) which has total of availability of 12 and 6 employed nurses respectively; (see table 1), an optimal total of 9 nurses for each of the wards in this group (F3) is needed in order to satisfy their daily requirement of 2 nurses. Also, x_1=1, x_2=1, x_3=1, x_4=1, x_6=1 indicates that one nurse should be assigned to Saturday – Sunday off, Sunday –Monday off, Monday –Tuesday off Tuesday –Wednesday off, and Thursday –Friday off respectively. (see table 9) Then, x_5=2 and x_7=2 implies that two nurses should be assigned to Wednesday-Thursday off and Friday-Saturday off.

In operating theatre and tuberculosis ward (F4) which has total availability of 2 and 1 employed nurses respectively; (see table 1), an optimal total of 2 nurses for each of the wards/unit in this group (F4) is needed in order to satisfy their daily requirement of 1 nurse. Also, x_1=1 x_5=1 indicates that one nurse should be assigned to Saturday –Sunday off, and Wednesday –Thursday respectively, (see table 9) Then, x_2=0, x_3=0 x_4=0, x_6=0 and x_7=0 implies that no nurse should be assigned to Sunday – Monday off, Monday –Tuesday off, Tuesday –Wednesday off, Thursday –Friday off and Friday-Saturday off.

These results are summarized in table 4 below

Table 5 Optimal solution for each ward in mile four hospital Abakaliki, Ebonyi State

Ward/units	Optimal allocation of Nurse for days off							Optimal number of Nurse required
	X1	X2	X3	X4	X5	X6	x7	
Post natal (1) and (2) ward	0	2	0	2	0	2	0	6
Post natal (3) ward	0	1	0	1	0	1	0	3
Nursery	0	2	0	2	0	2	0	6
Labour ward	1	1	1	1	2	1	2	9
Antenatal clinic (A N C)	1	1	1	1	2	1	2	9
Anaesthesia unit (A R T)	0	1	0	1	0	1	0	3
Multiple drug resistant (MDR)	0	1	0	1	0	1	0	3
Out-patient department (OPD)	0	1	0	1	0	1	0	3
Children ward	0	2	0	2	0	2	0	6
Operating theatre	1	0	0	0	1	0	0	2
Admin / Counselling unit	0	1	0	1	0	1	0	3
Tuberculosis ward	1	0	0	0	1	0	0	2

Table 6 Sample of one week Roster for nurse in F1 wards/units

Nurse ID number	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	Off	On	On	On	On	On	Off
2	Off	On	On	On	On	On	Off
3	On	Off	Off	On	On	On	On
4	On	Off	Off	On	On	On	On
5	On	On	On	Off	Off	On	On
6	On	On	On	Off	Off	On	On
Required	4	4	4	4	4	4	4

Assigned	4	4	4	4	4	6	4
Excess	0	0	0	0	0	0	0

Table 7 Sample of one week Roster for nurse in F2 wards/units

Nurse ID number	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	Off	On	On	On	On	On	Off
2	On	Off	Off	On	On	On	On
3	On	On	On	Off	Off	On	On
Required	2	2	2	2	2	2	2
Assigned	2	2	2	2	2	3	2
Excess	0	0	0	0	0	1	0

Table 8 Sample of one week Roster for nurse in F3 wards/units

Nurse ID number	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	On	On	On	On	On	Off	Off
2	Off	On	On	On	On	On	Off
3	Off	Off	On	On	On	On	On
4	On	Off	Off	On	On	On	On
5	On	On	Off	Off	On	On	On
6	On	On	On	Off	On	On	On
7	On	On	On	Off	Off	On	On
8	On	On	On	On	Off	Off	On
9	On	On	On	On	Off	Off	On
Required	6	6	6	6	6	6	6
Assigned	7	7	6	6	6	6	7
Excess	1	1	0	0	0	0	1

Table 9 Sample of one week Roster for nurse in F4 wards/units

Nurse ID number	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	On	On	On	On	On	Off	Off
2	On	On	Off	Off	On	On	On
Required	1	1	1	1	1	1	1
Assigned	2	2	1	1	2	1	1
Excess	1	1	0	0	1	0	0

Table 5, 6,7, 8, and 9 shows the resulting work schedule for the wards/unit with ON representing assignments and OFF representing day–off. The last three rows of each table show the nursing staff requirements, total number of assignments and excess in a given day.

5. Conclusion

This study successfully applies integer programming (IP) and the branch and bound (B and B) algorithm to optimize nurse scheduling in mile four hospital Abakaliki, ensuring an efficient and fair workforce allocation. The developed mathematical model systematically assigns nurses to shifts while meeting daily staff requirements, balancing workload distribution, and ensuring compliance with labor regulations. In this work, we have determined an efficient and fair scheduling for the four groups, each nurse off duty twice and on duty five times a week. The optimal nurse shift scheduling to is to allocate 6 nurses for group F1, 3 nurses for group F2, 9 nurses for group F3 and 2 nurses for group F4. This minimizes the total nursing staff cost by 0.09%, with F1 group contributing 0.05%, F2 group contributing 0.05%, F3 group contributing 0% and F4 group contributing -0.01%. The optimal schedule ensures continuous patient care without overburdening staff.

Recommendations

This work provides a strong foundation for data driven work in hospitals. Future research could explore

- Incorporating real time adjustments for emergencies and varying patient loads.
- Considering nurse preferences, fatigue levels and patients' acuity.
- Machine learning can also be integrated to predict nurse schedules based on historical hospital data.

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest to be disclosed.

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